

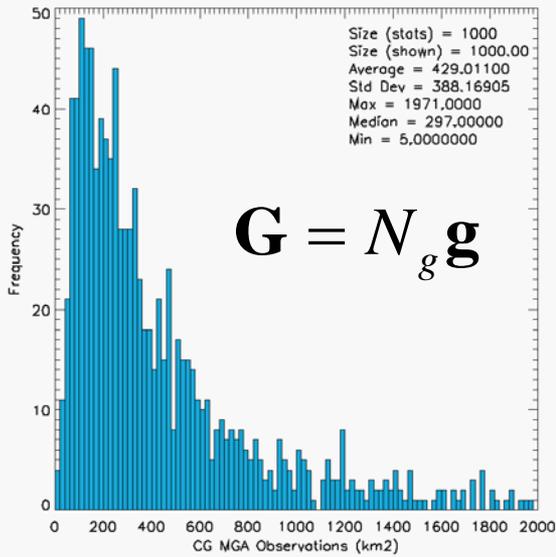
# Ground Flash Fraction Retrieval Algorithm

GLM Science Meeting  
Huntsville, AL  
September 24, 2013

Dr. William Koshak, NASA-Marshall Space Flight Center  
Dr. Richard Solakiewicz, Chicago State University

# Summary of Main Advances

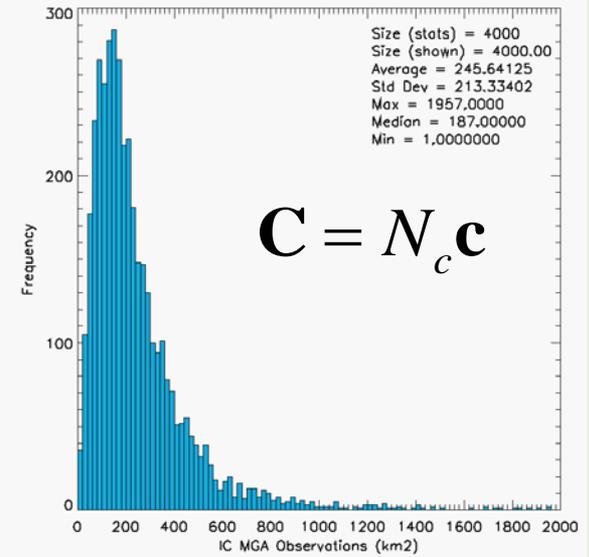
- **Mathematical Advances to Analytic Perturbation Method**
  - Derived closed form solution for ground flash fraction ... no numerical scanning of H function.
  - Derived explicit analytic forms for the perturbation vectors.
  - **Derived a way to determine flash type of specific flashes.**
- **APM Error Analysis Completed Via Simulations**
- **DEMO Completed**
- **Journal Paper Completed & Ready for Submission (JTECH)**



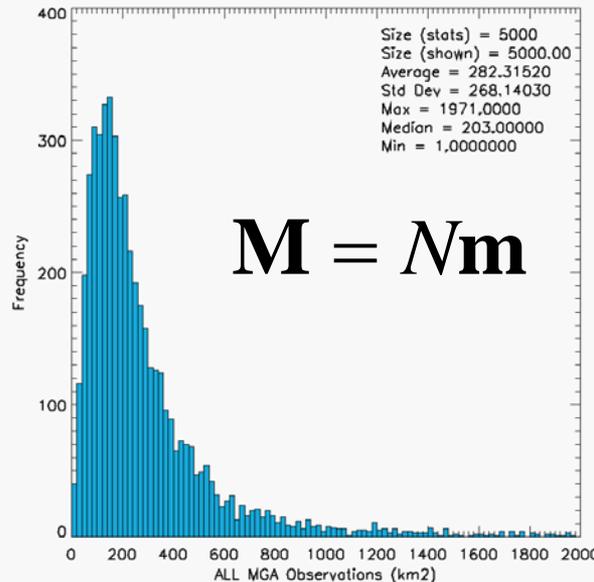
$$\mathbf{G} = N_g \mathbf{g}$$



**MIX**



$$\mathbf{C} = N_c \mathbf{c}$$



$$\mathbf{M} = N \mathbf{m}$$

$$\mathbf{M} = \mathbf{G} + \mathbf{C}$$

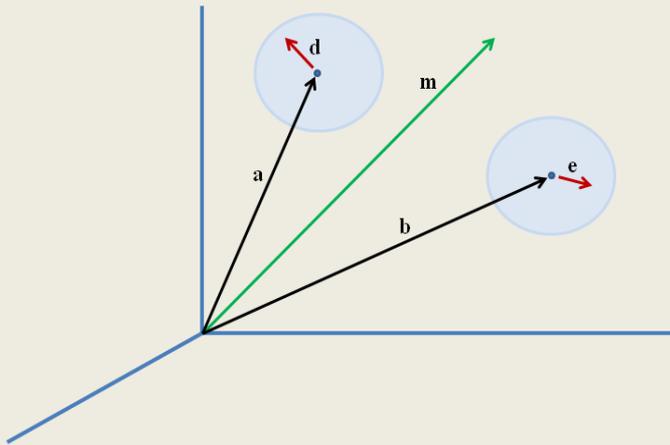
$$\mathbf{m} = \alpha \mathbf{g} + (1 - \alpha) \mathbf{c}$$

$$\alpha = N_g / N$$

Instrument Observes  
 $N$  MGAs.

**Invert** this Distribution  
 to Find Ground Flash  
 Fraction  $\alpha$

# APM SOLUTION PROCESS

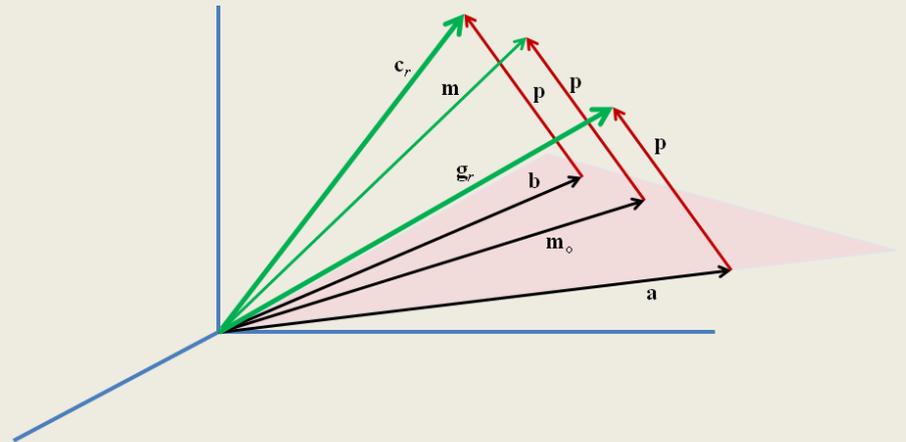


Geometry of the APM depicted in 3-space. Climate vectors are  $(\mathbf{a}, \mathbf{b})$  which are perturbed by  $(\mathbf{d}, \mathbf{e})$  so that the estimates  $\mathbf{g} \approx \mathbf{a} + \mathbf{d}$ ,  $\mathbf{c} \approx \mathbf{b} + \mathbf{e}$  are made.

$$\text{Minimize: } H = \alpha_r \mathbf{d}^2 + (1 - \alpha_r) \mathbf{e}^2$$

$$\text{Subject to: } \mathbf{m} = \alpha_r \mathbf{g}_r + (1 - \alpha_r) \mathbf{c}_r$$

$$\rightarrow \text{Results in perturbation: } \mathbf{p} = \mathbf{m} - \alpha_r \mathbf{a} - (1 - \alpha_r) \mathbf{b}$$



The effect of the perturbation is to create a hyperplane out of  $(\mathbf{g}_r, \mathbf{c}_r)$  that contains the (fixed) mixture density  $\mathbf{m}$ . The geometry is depicted in 3-space.

$$\mathbf{m} = \alpha \mathbf{g} + (1 - \alpha) \mathbf{c} \quad (\text{truth})$$

$$\mathbf{m} = \alpha_r \mathbf{g}_r + (1 - \alpha_r) \mathbf{c}_r \quad (\text{model})$$

# APM SOLUTION SUMMARY

**PERTURBATION GIVES:**

$$\alpha_r = \frac{(\mathbf{m} - \mathbf{b})^T (\mathbf{a} - \mathbf{b})}{(\mathbf{a} - \mathbf{b})^2}$$

$$\mathbf{g}_r = \mathbf{m} + (1 - \alpha_r)(\mathbf{a} - \mathbf{b})$$

$$\mathbf{c}_r = \mathbf{m} - \alpha_r(\mathbf{a} - \mathbf{b})$$

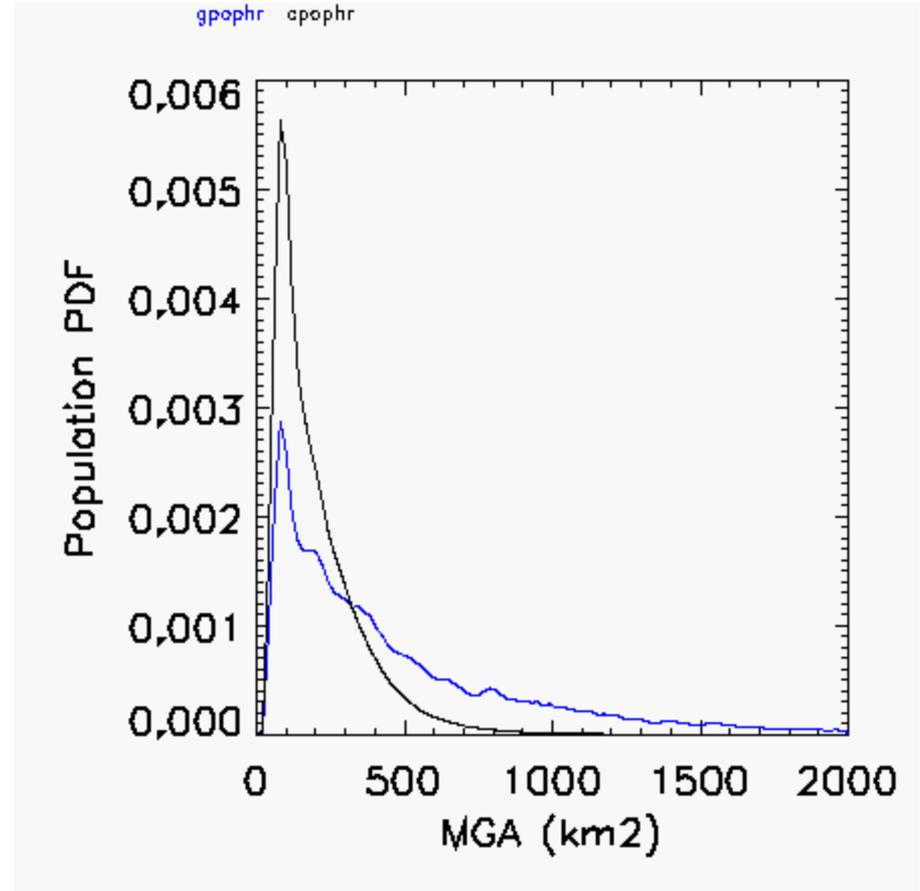
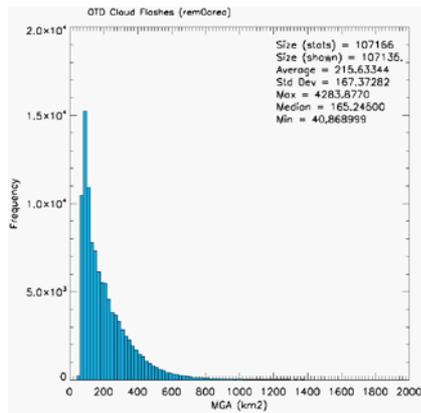
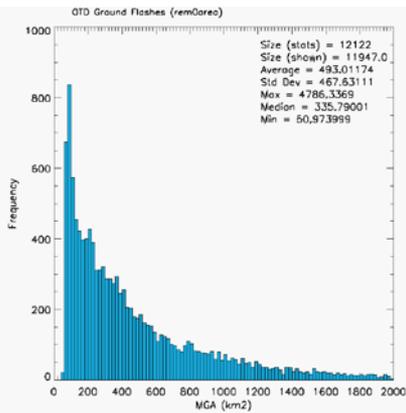
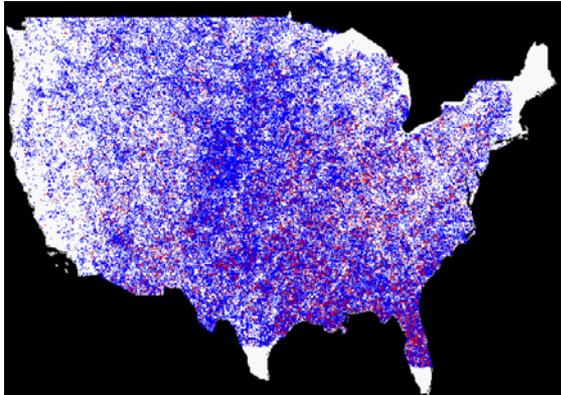
**PROBABILITY FLASH, HAVING MGA =  $x$ , IS A GROUND FLASH:**

$$P_g(x) = \frac{\alpha_r g_r(x)}{\alpha_r g_r(x) + (1 - \alpha_r)c_r(x)}$$

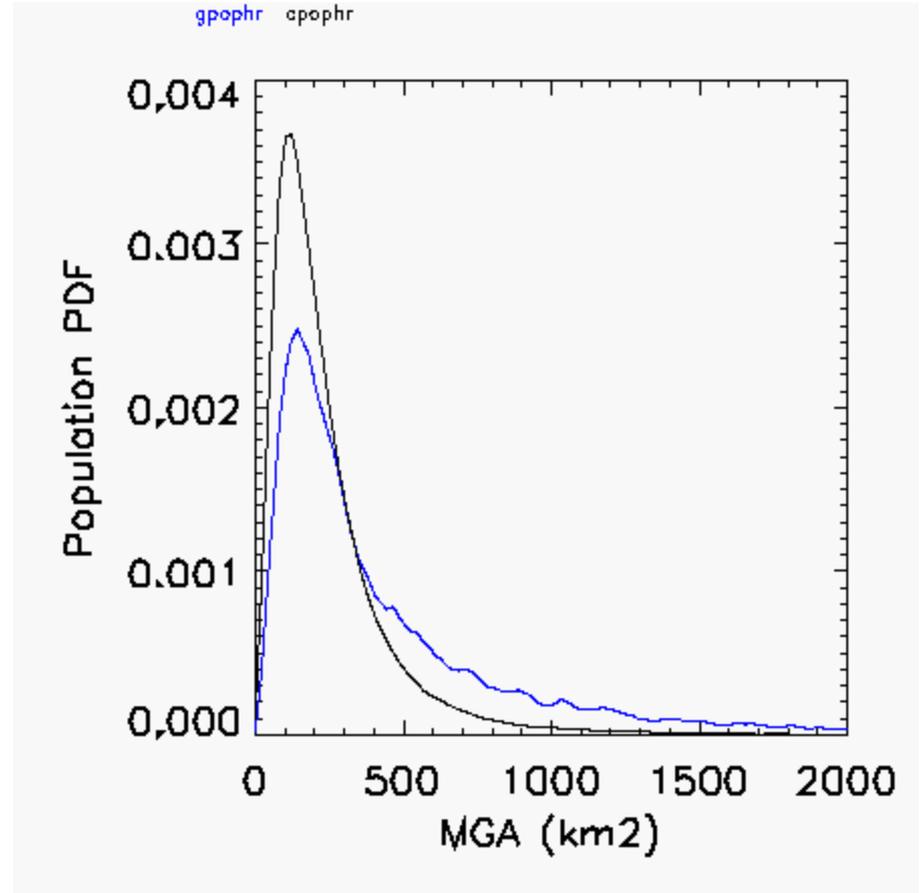
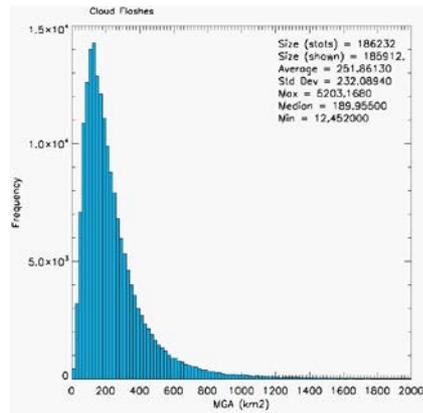
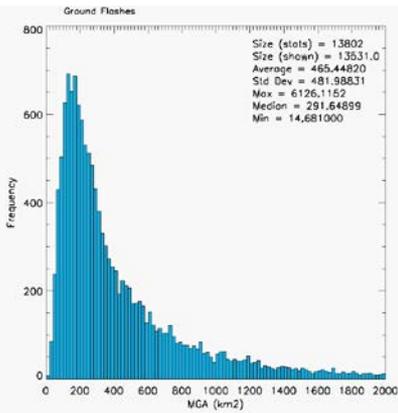
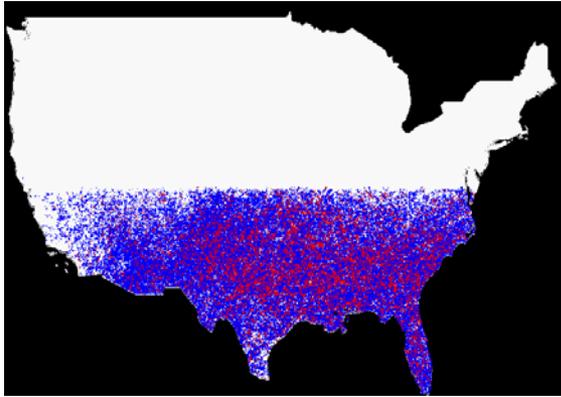
$$P_g(x) > 0.5 \Rightarrow \text{Ground Flash}$$

$$P_g(x) \leq 0.5 \Rightarrow \text{Cloud Flash}$$

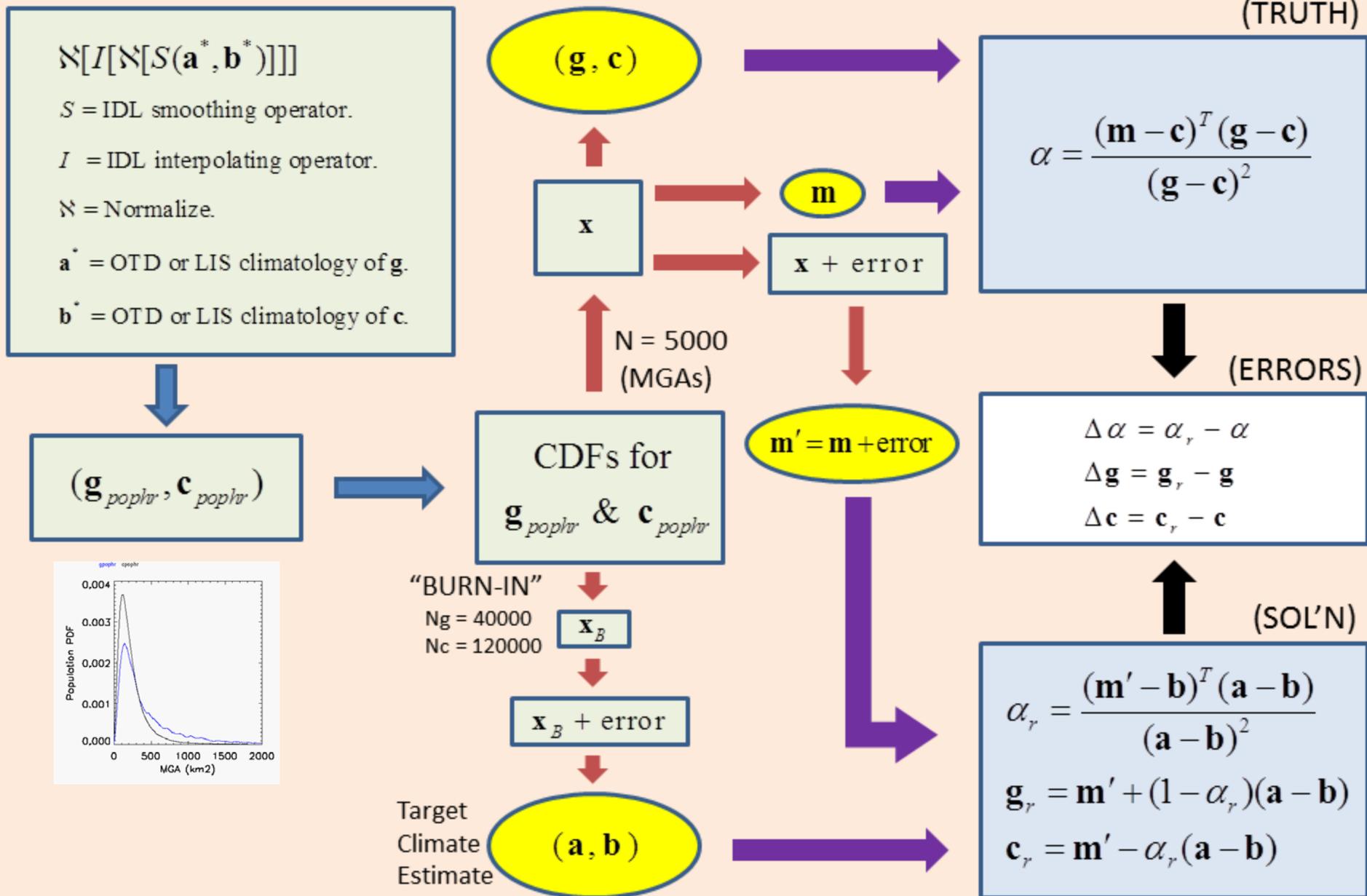
# Simulate Population from OTD Data (Flash Type Categorized by NLDN)



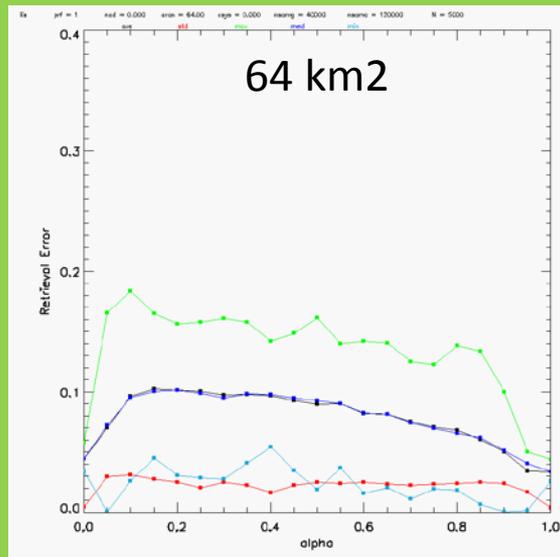
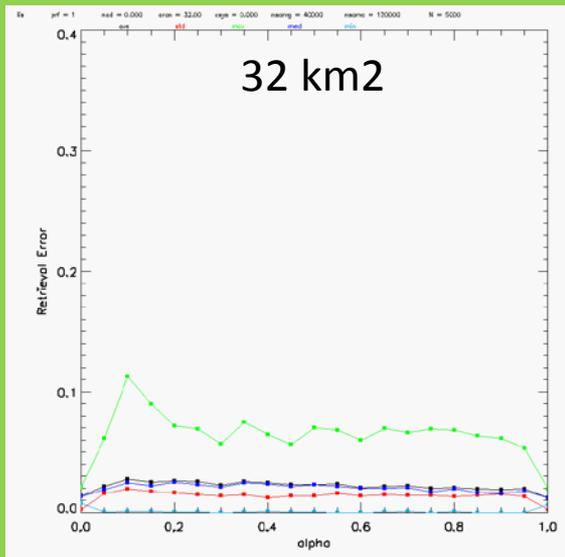
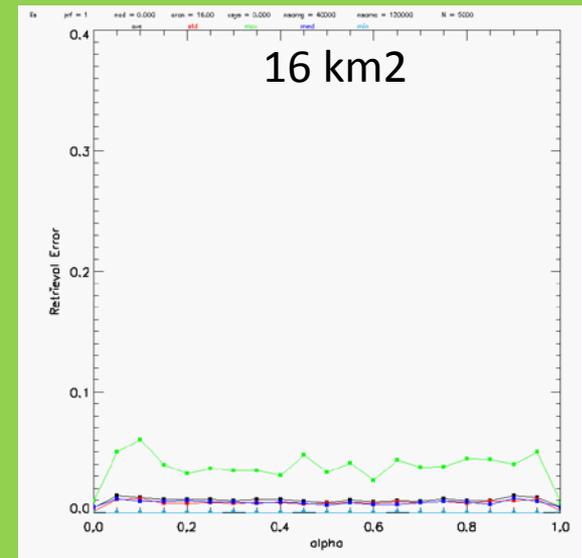
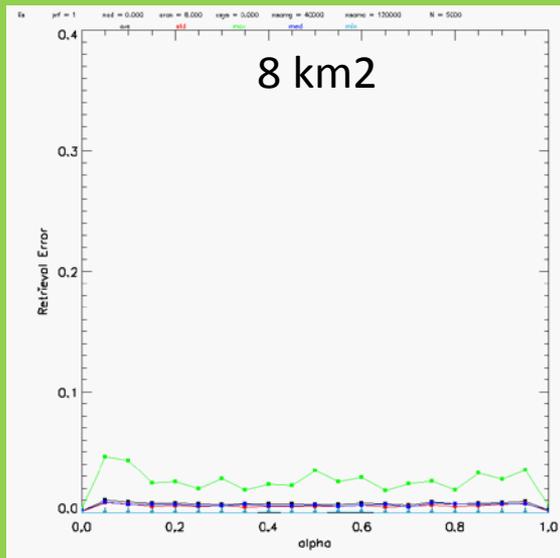
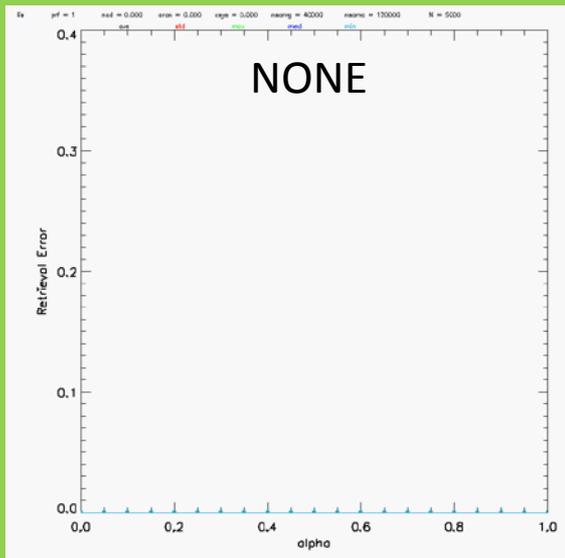
# Simulate Population from LIS Data (Flash Type Categorized by NLDN)



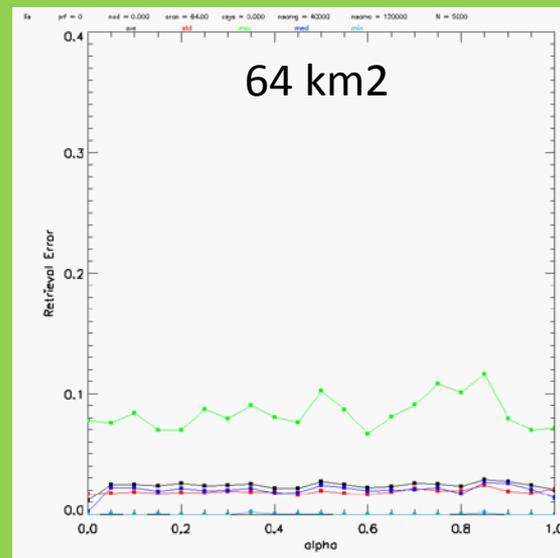
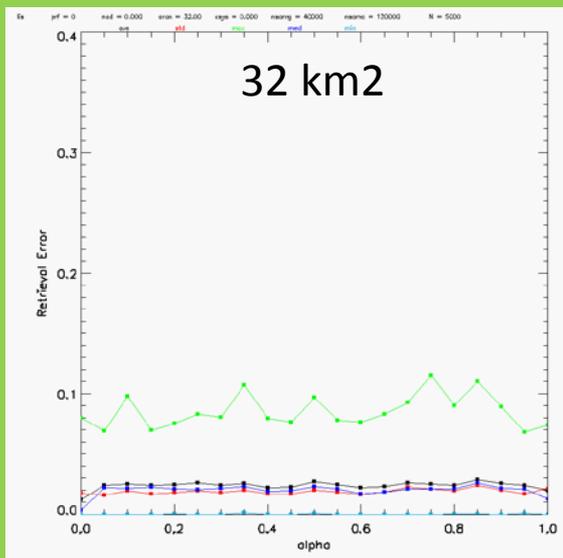
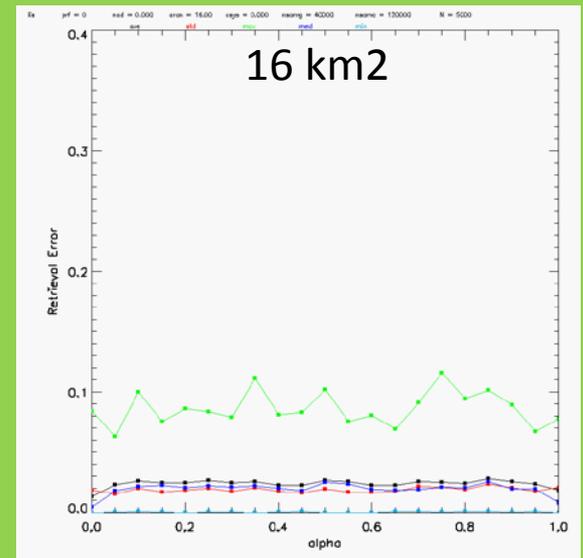
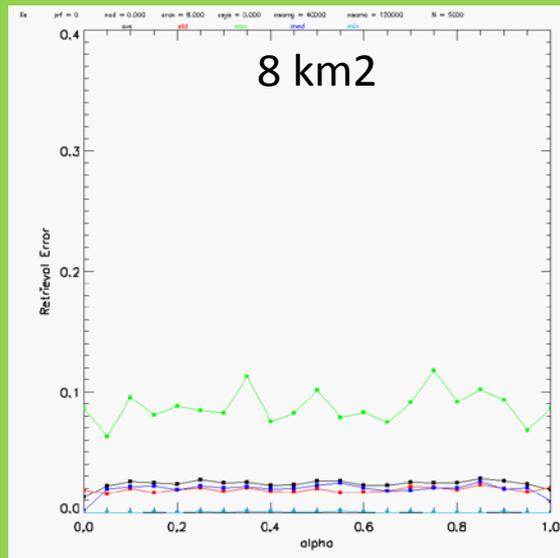
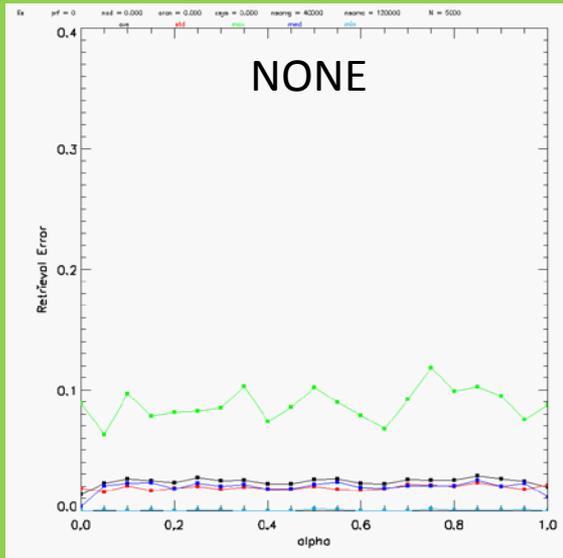
# Inversion Algorithm Performance Simulator



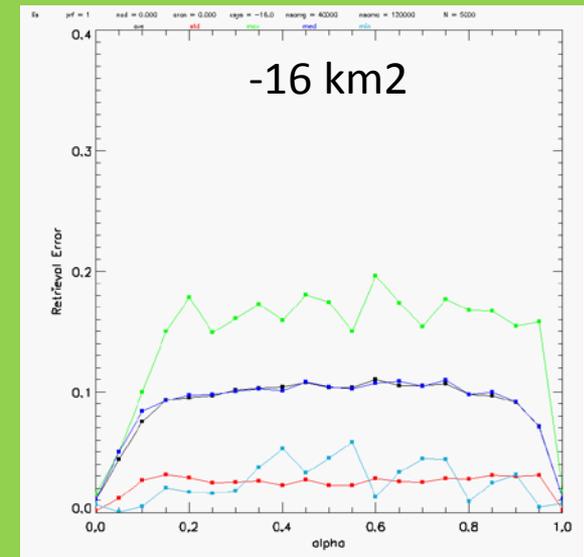
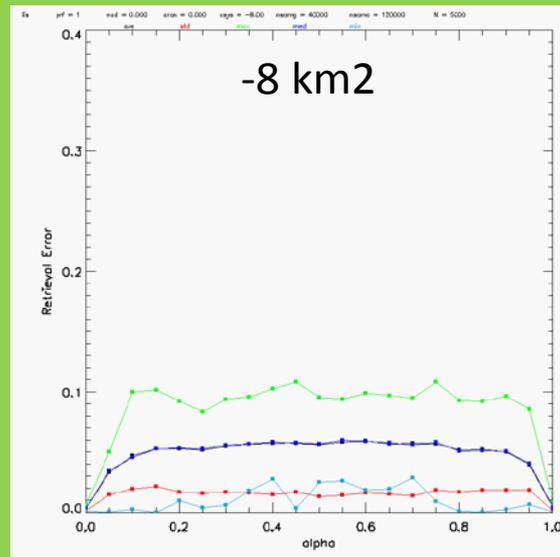
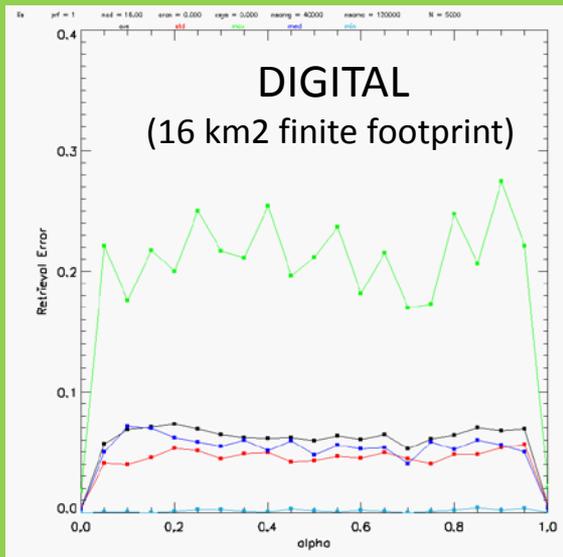
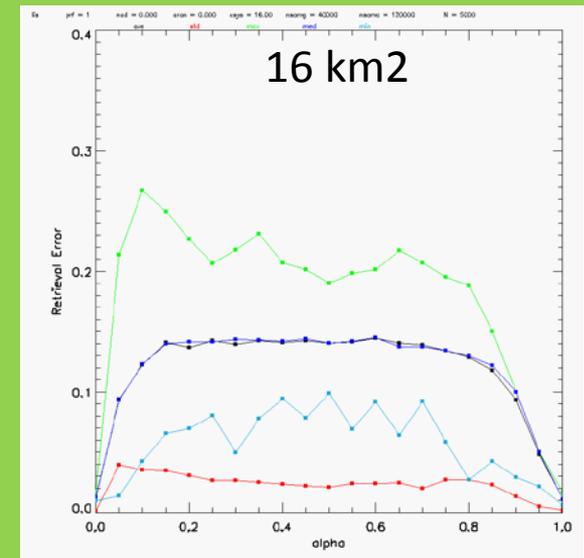
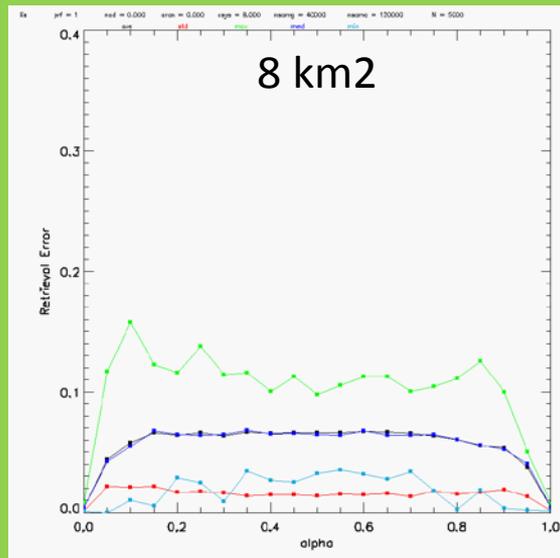
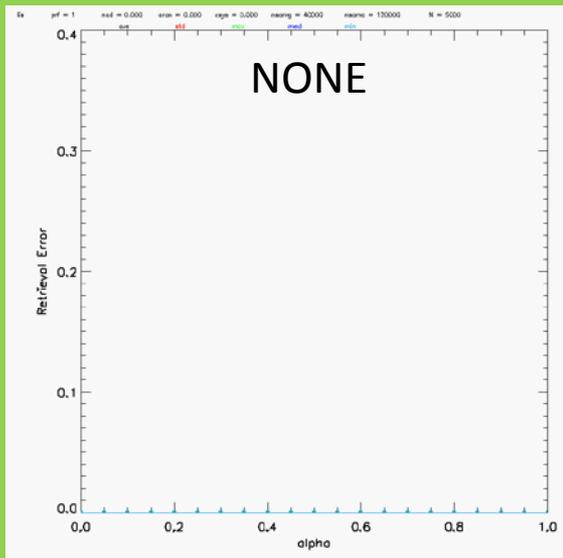
# EFFECT OF RANDOM ERROR (a = g, b = c)



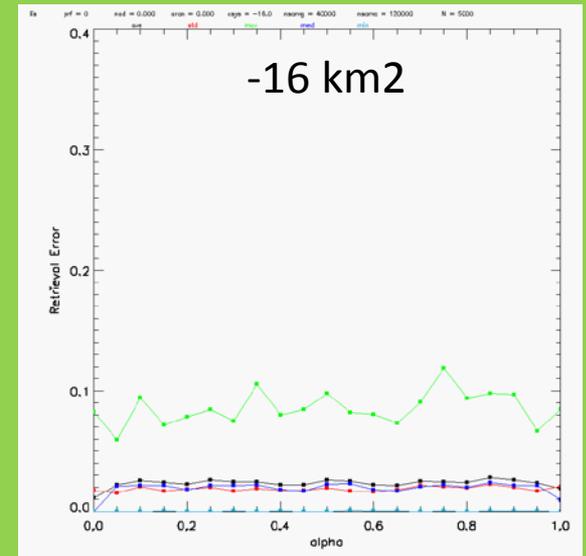
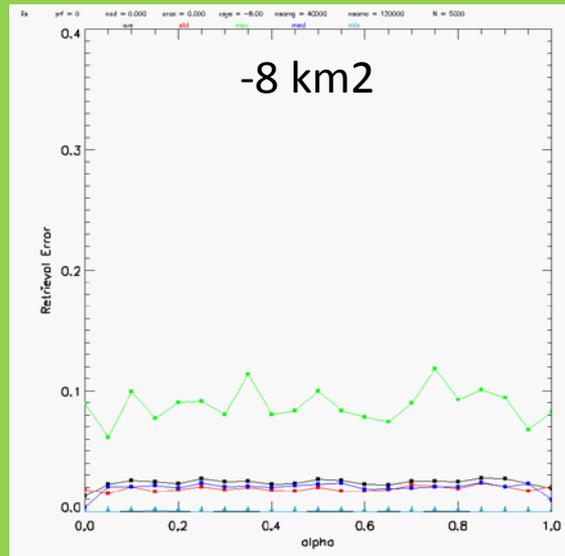
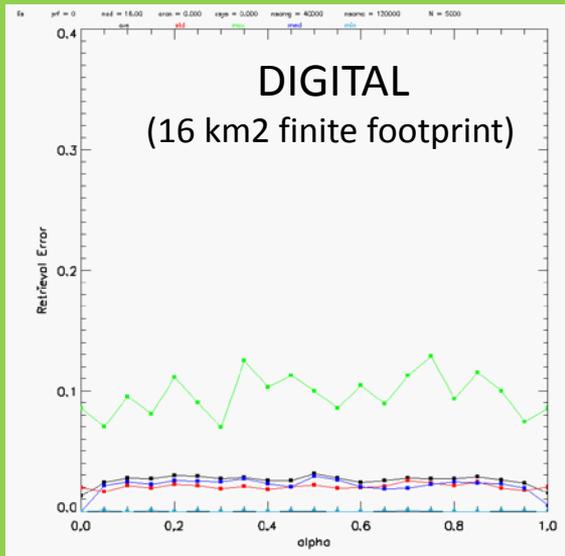
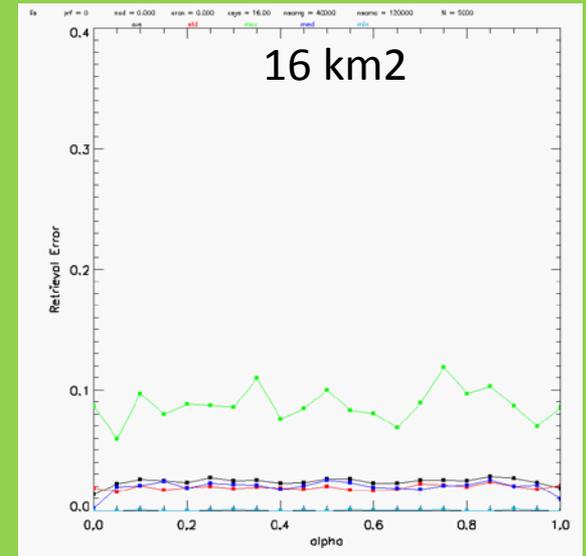
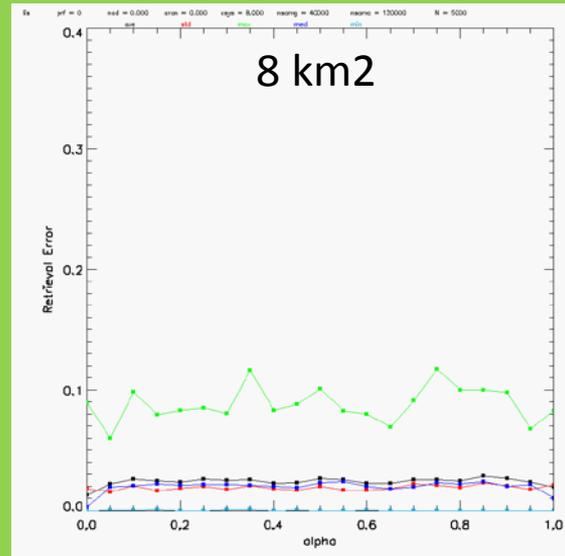
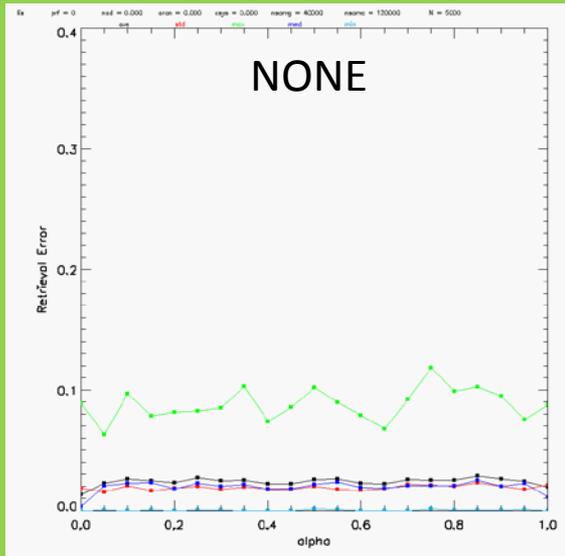
# EFFECT OF RANDOM ERROR (a and b from method)



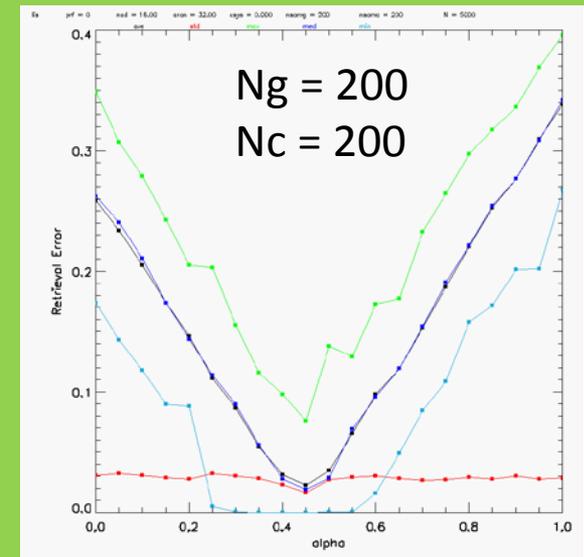
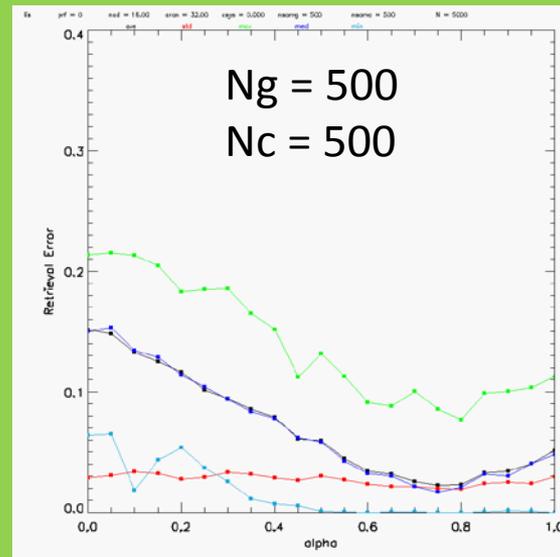
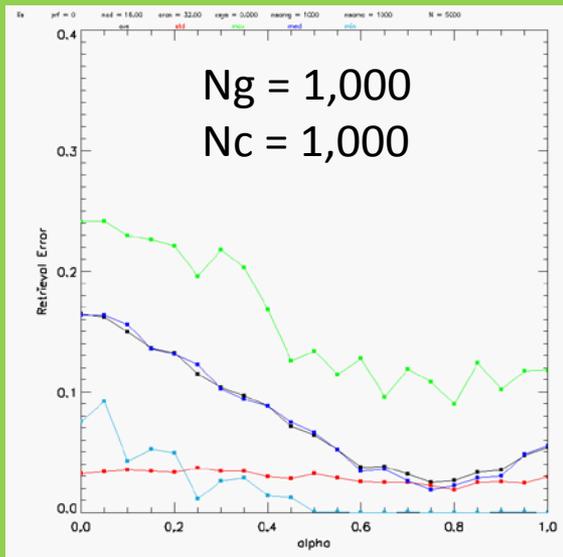
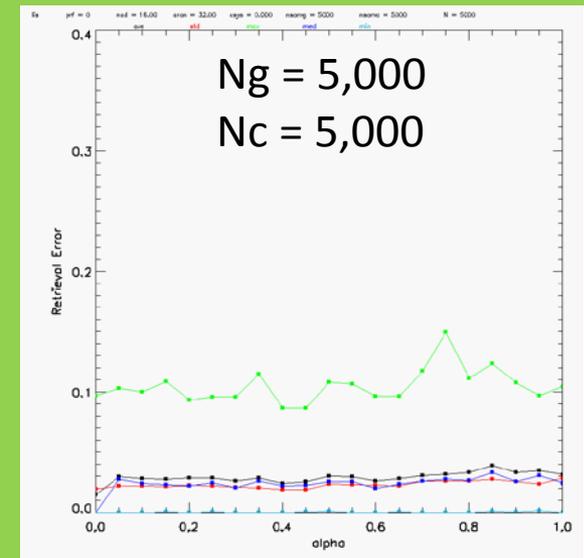
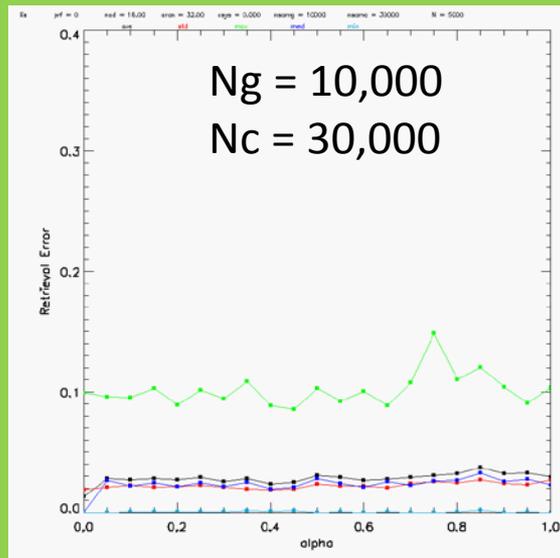
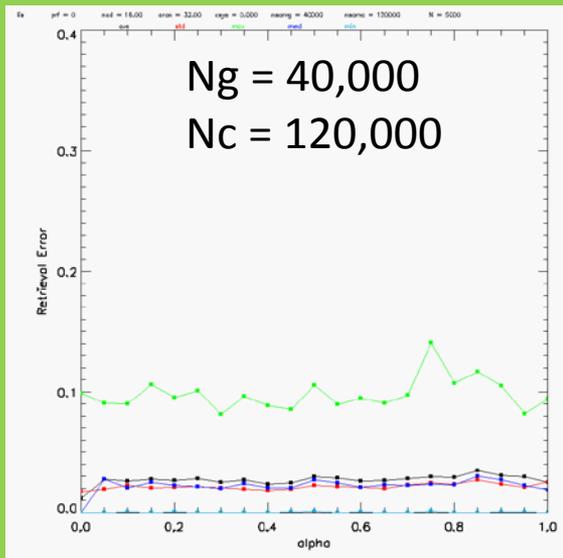
# EFFECT OF SYSTEMATIC ERROR ( $a = g, b = c$ )



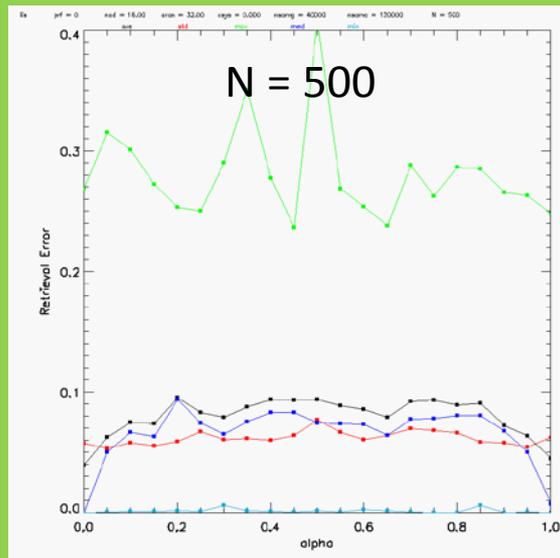
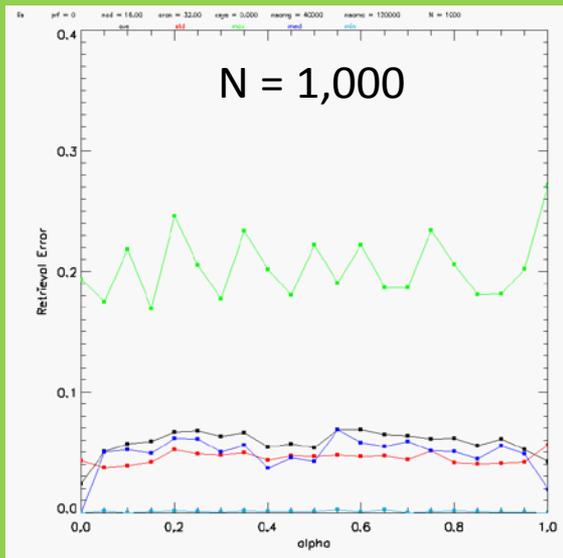
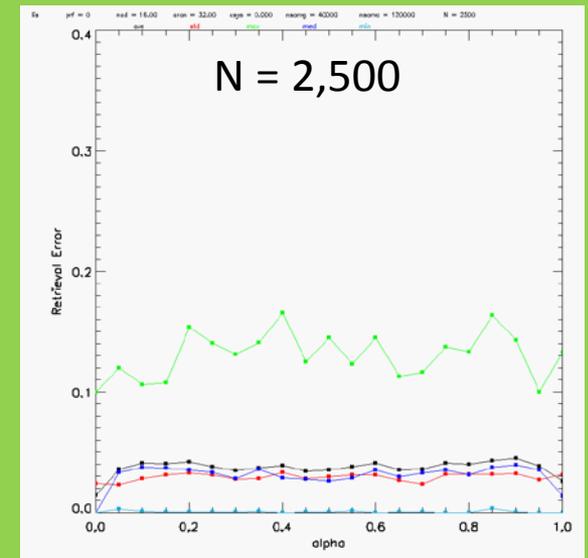
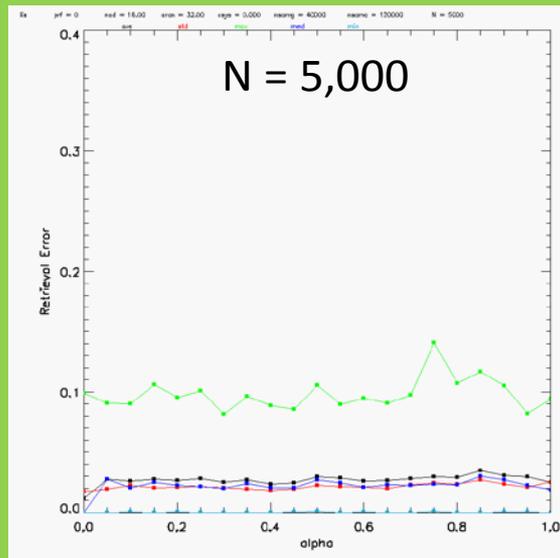
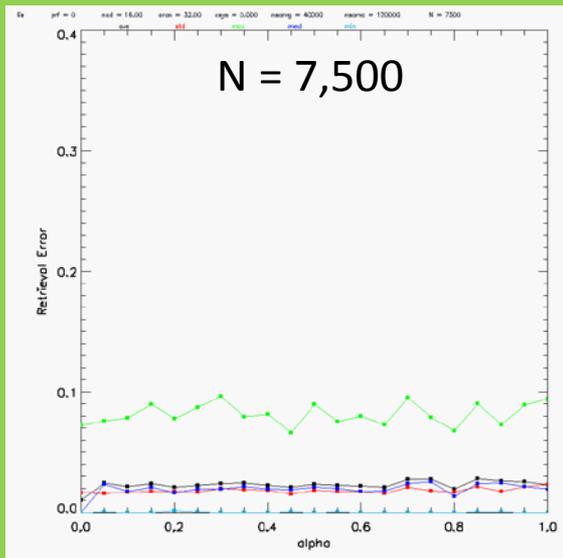
# EFFECT OF SYSTEMATIC ERROR (a and b from method)



# EFFECT OF CLIMATOLOGY SAMPLE SIZE



# EFFECT OF OBSERVATION SAMPLE SIZE $N$



# NOMINAL (LIS data)

# Observations:

N = 5000

# in Climate Sampling:

Ng = 40,000

Nc = 120,000

Instr. Measurement Error:

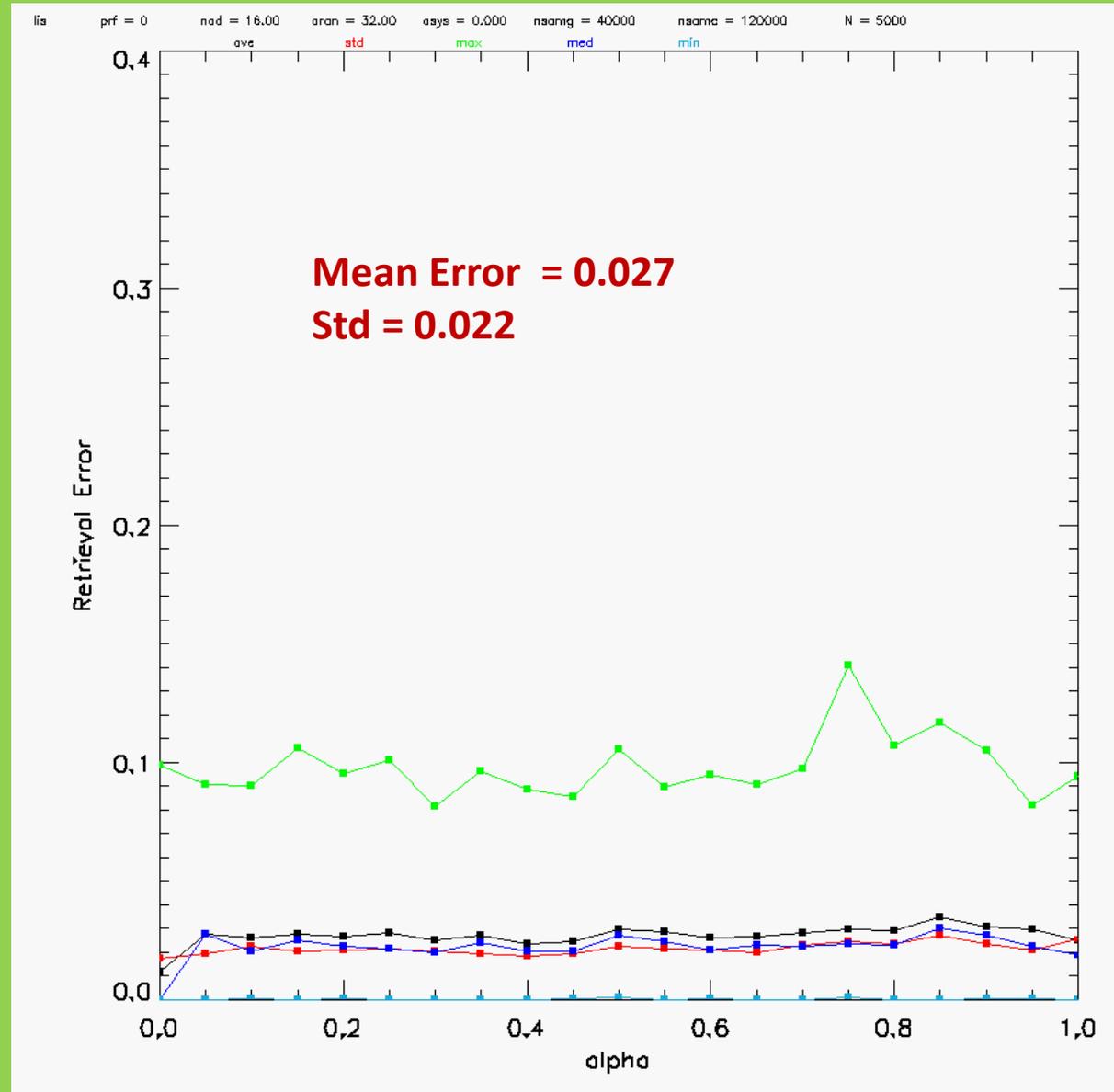
Random =  $\pm 32$  km<sup>2</sup>

Systematic = 16 km<sup>2</sup> digital

gpop mean = 465.4 km<sup>2</sup>

cpop mean = 251.9 km<sup>2</sup>

difference = 213.5 km<sup>2</sup>



# NOMINAL (OTD data)

## # Observations:

N = 5000

## # in Climate Sampling:

Ng = 40,000

Nc = 120,000

## Instr. Measurement Error:

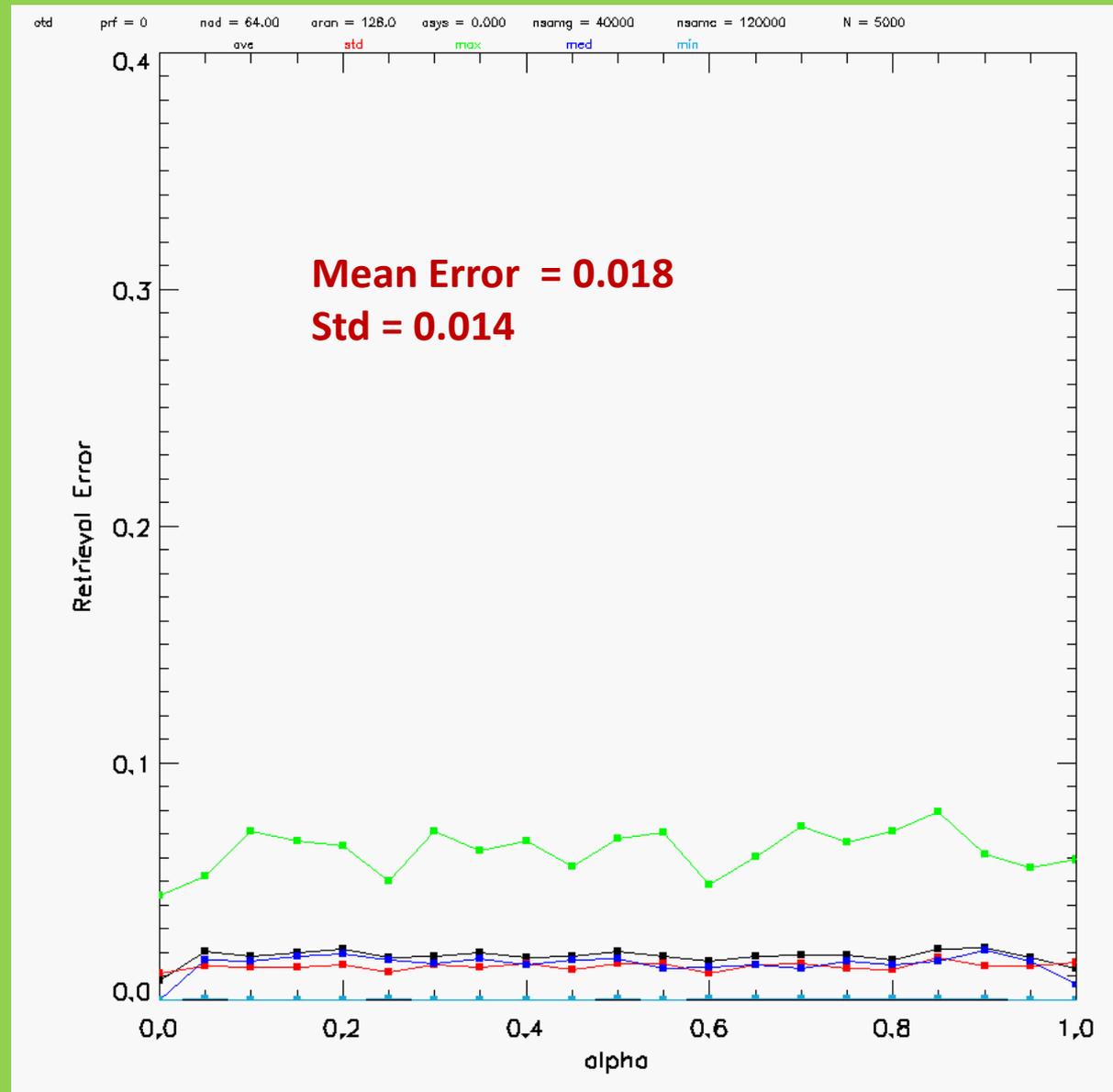
Random =  $\pm 128$  km<sup>2</sup>

Systematic = 64 km<sup>2</sup> digital

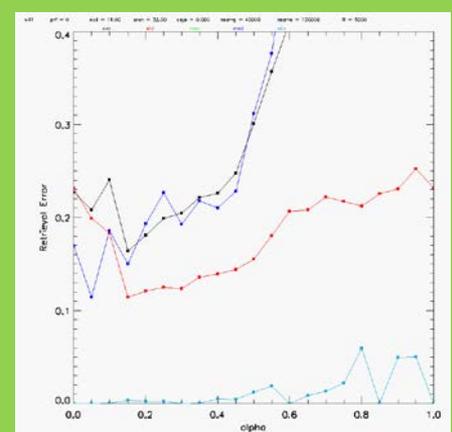
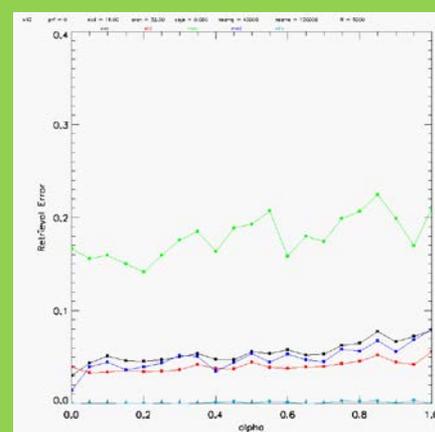
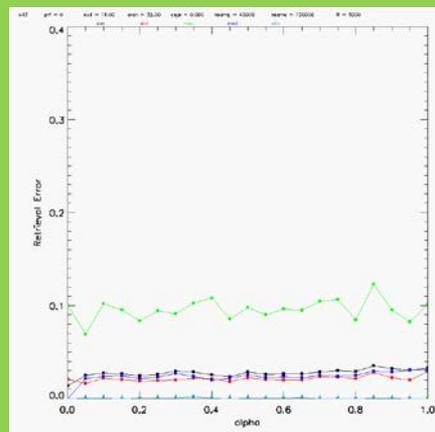
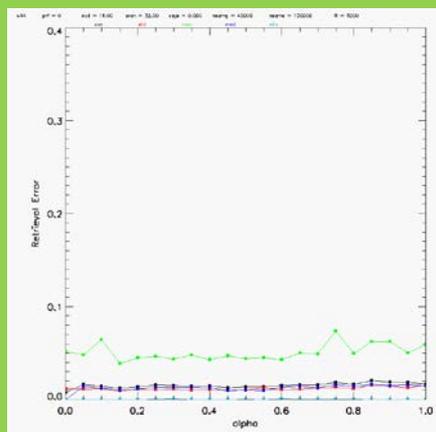
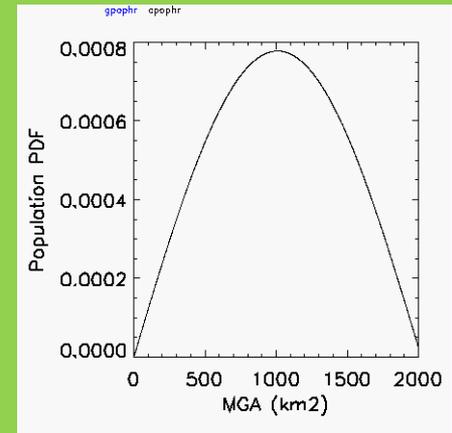
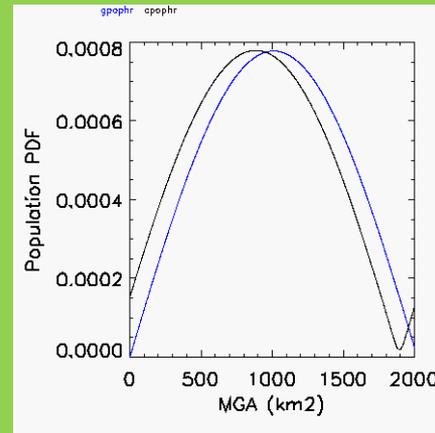
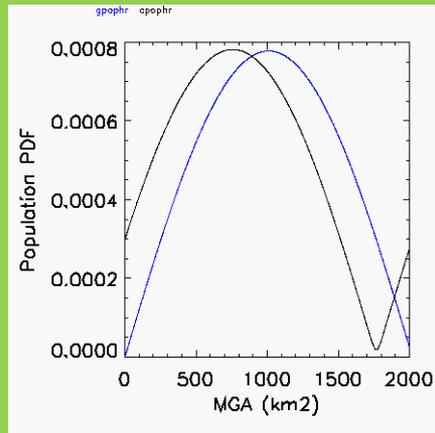
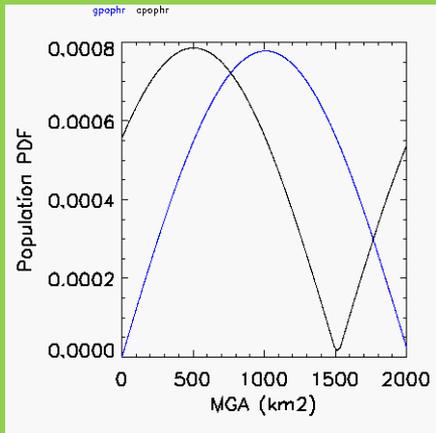
gpop mean = 493.0 km<sup>2</sup>

cpop mean = 215.6 km<sup>2</sup>

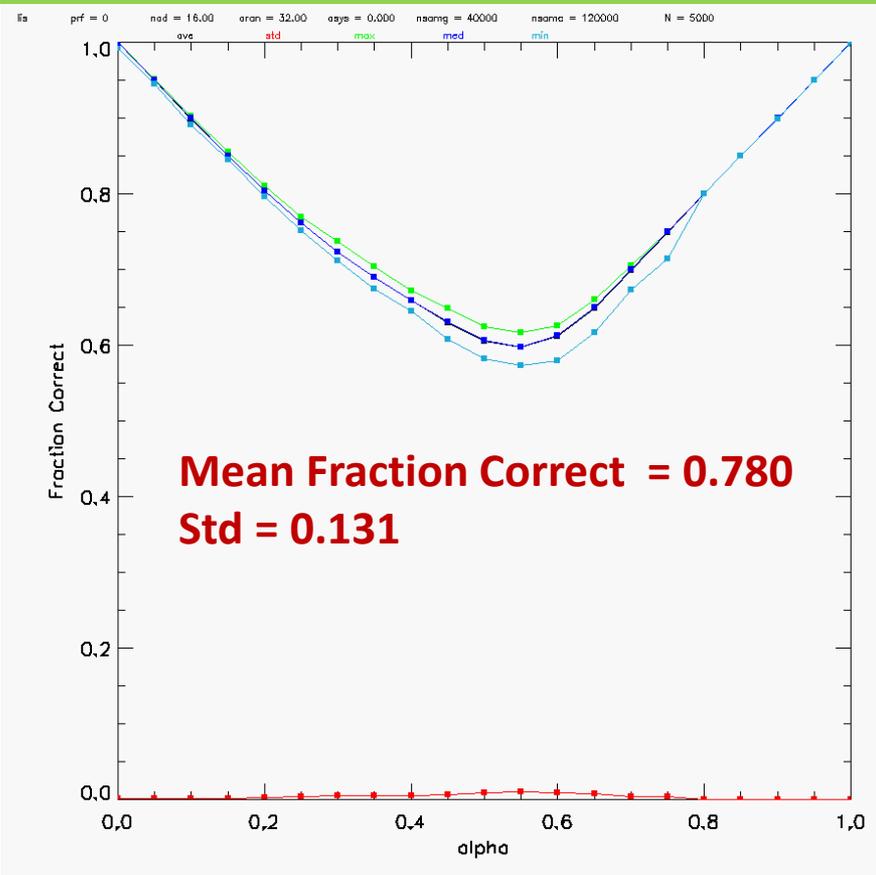
difference = 277.4 km<sup>2</sup>



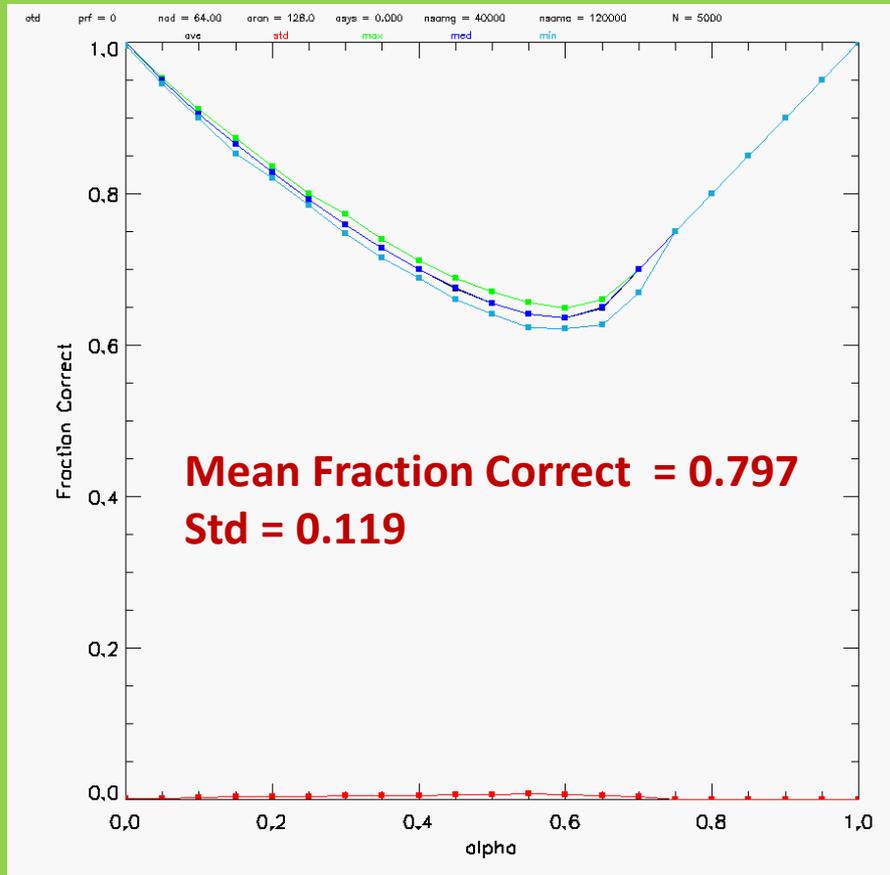
# CAN HANDLE ARBITRARY DISTRIBUTIONS (AS USUAL, CG & IC DISTRIBUTIONS NEED TO BE DISTINCT)



# PERFORMANCE OF SPECIFIC FLASH TYPING

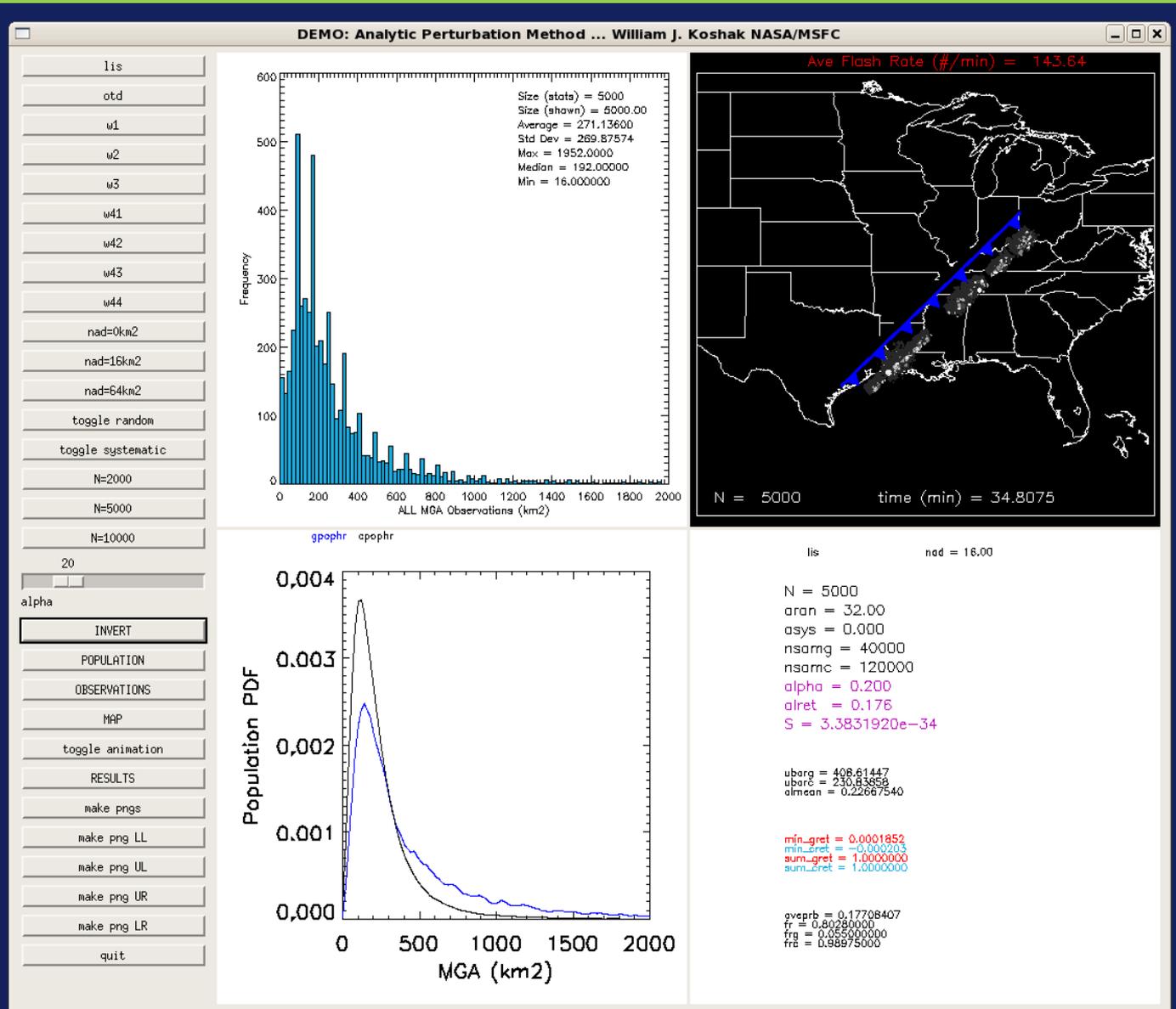


LIS data



OTD data

# FUNCTIONAL DEMO NOW EXISTS



**Thank You**